IX. *On the Proper Motion of the Solar System.*

By THOMAS GALLOWAY, *Esq., M.A., F.R.S., Sec. R.A.S.*

Received March 4,—Read April 15, 1847.

THE third volume of the Mémoires presentés par divers Savans of the Imperial Academy of St. Petersburg, published in 1837, contains a paper by Professor ARGELANDER, in which that distinguished astronomer has discussed the question of the proper motion of the solar system, and determined the probable situation in space of the point towards which the sun is at present advancing. This determination was founded on the proper motions of 390 stars situated between the north pole and the tropic of Capricorn, as shown by a comparison of their positions in 1775 according to BRADLEY's observations, reduced by BESSEL, with their positions in 1830 computed from the observations made by ARGELANDER himself at Abo; every star being taken into account which appeared to have a proper motion amounting to a tenth of a second in space annually. Two other investigations of the same question have since been published; one by LUNDAHL, founded on the proper motions of 147 stars, as shown by a comparison of the observations of BRADLEY and POND, and the other by OTTO STRUVE, based on 392 stars, whose proper motions were determined by a comparison of BRADLEY's observations with those made at the observatory of Dorpat. From these three investigations the direction of the sun's motion in space may be considered, perhaps, to have been determined with as great an approximation to accuracy as can be attained in the present state of our knowledge of the proper motions of the stars in the northern hemisphere. The recent catalogues of Mr. JOHNSON and the late Professor HENDERSON, deduced from the observations made by those astronomers respectively at St. Helena and the Cape of Good Hope, on being compared with the Cape observations of LACAILLE made about the middle of the last century, show that a considerable number of the southern stars have also very appreciable proper motions; and it appeared to me to be a matter of some interest to inquire whether the proper motions so determined afford any confirmation of the results obtained by ARGELANDER, LUNDAHL and STRUVE, or favoured the hypothesis of a displacement of the solar system. The result of this inquiry I have now the honour of submitting to the Royal Society, in whose Transactions the existence of relative displacements among the fixed stars was first announced, and the probable direction of the sun's motion first indicated. Independently of theoretical considerations, the subject is of much importance in astronomy. The proper motions of the stars, which may be said to be the only residual astronomical phenomena now remaining to be accounted for by theory, mix themselves up with the determination of the precession and other fundamental elements; and the first step towards acquiring any knowledge of their laws, quantities, or directions, is obviously to distinguish between what is real and what is only apparent, and to separate from the whole observed displacement the effect due to the motion of our own system.

Before proceeding to describe the data and results of the present investigation, it will be desirable, perhaps, to give a brief notice of the principal inquiries that have heretofore been undertaken with reference to the same subject.

In the Philosophical Transactions for 1713, HALLEY first called attention to the circumstance that a comparison of the ancient with modern observations showed that three of the principal stars, Sirius, α Tauri and Aldebaran, had changed their positions relatively to the fixed circles of the sphere, and advanced considerably towards the south. Until this time the notion had universally prevailed that the places of the stars are subject to no relative change; but no sooner was the notion called in question than instances of such change were multiplied; and the proper motions of stars being once admitted, it was naturally suggested that the sun itself partakes of a similar motion. BRADLEY, in the memorable paper in which he announced the discovery of the nutation, published in the Philosophical Transactions for 1748, described the appearances which would result from a change of the position of the solar system in absolute space, but he made no attempt to explain the observed phenomena on this hypothesis, and remarked that the alterations in the relative positions of the stars might arise from so great a variety of causes, that many centuries, perhaps, would be required to discover their laws.

TOBIAS MAYER, in a memoir presented to the Göttingen Society in 1760, and published among his Opera Inedita in 1775, gave a list of eighty stars which had been observed by RŒMER in 1706, and compared their places as given by RŒMER with those deduced from his own observations (1756) and those of LACAILLE (1750). Out of the eighty stars about fifteen or twenty were found in respect of which the difference of position, either in right ascension or declination, exceeded 15″, a quantity which he considered would be at least equal to the error of observation. In the cases, therefore, in which the difference did not much exceed 15″, he thought a proper motion was not improbable; but in some cases, as those of Arcturus, Sirius, Procyon, α Aquilæ, Piscis Austrinus, and a few others, the difference was so great that there could be no question about the existence of such motion. MAYER also made the remark, that the stars which appear to change their places are not confined to those of the first or second order of magnitude, which, by reason of their greater brilliancy, might be presumed to be the nearest to the earth; and that among the brighter there are some which appear to be altogether at rest. And he further remarked, that although it is by no means improbable that the sun as well as the stars may have a motion of its own, yet, as the observed changes of position do not follow the law they would observe if caused by the motion of our solar system towards a given point in the heavens, it is manifest they do not proceed from this or any similar common cause, but belong to the stars themselves, though the true and genuine cause may remain unknown for ages.

This conclusion, if understood as applying to the whole of the changes of position indicated by the comparison of the catalogues, was no doubt correct; but it is evident that, although the apparent displacements may not be capable of being completely explained on the hypothesis of a solar motion, it by no means follows that they do not in part depend upon this cause, and that, widely as the observed motions may differ in their relative directions, there may not still be a preponderance of motion, or a general tendency to move, towards some determinate point. LAMBERT, writing in 1761, remarked that the apparent changes in the positions of the fixed stars depend on the motion of the sun as well as on the motions of the stars themselves, whence, he says, "we may perhaps in time arrive at the means of determining towards what region of space the sun holds its course." The same philosopher also noticed that the rotation of the sun on its axis gives rise to a probability of its translation in space, although no proof can be given that the latter motion is a necessary consequence of the former.

The probability of the opinion that the observed proper motions of the stars are compounded of a real and an apparent motion was also noticed by MICHELL, who, in a note to a paper published in the Philosophical Transactions for 1767, remarked that the apparent change of situation which has been observed in a few of the stars, is a strong circumstance in favour of the opinion that those stars are among the nearest to us; and that the apparent displacement may be owing either to a real motion of the stars themselves, or to that of the sun, or partly to the one and partly to the other. And, he adds, "as far as it is owing to the sun's motion it may be regarded as a kind of secular parallax, which, if the annual parallax of a few of the stars should some time or other be discovered, and the quantity and direction of the sun's motion should be discovered also, might serve to inform us of the distance of many of them, which it would be utterly impossible to find out by any other means."

LAMBERT's argument, that the fact of the sun's rotatory motion about its axis affords a presumption of its translation in space, was adopted by LALANDE, who, in a memoir presented to the Academy of Sciences of Paris in 1776, concludes that inasmuch as the application of any force causing a body to turn about its centre cannot fail to displace the centre, the sun must necessarily have a real motion in absolute space. This argument will not be allowed to have much weight when it is considered that the sun's rotatory motion *may*, and probably does, proceed from causes wholly different from an eccentric impulsion.

Whatever degree of probability such *à priori* considerations may be supposed to give to the hypothesis of the sun's proper motion, it is evident that something more is necessary to render the hypothesis of any practical importance. The first astronomer who attempted to prove the existence of the sun's motion from observations, and ventured to assign the precise point in the heavens towards which our system is actually borne, was Sir WILLIAM HERSCHEL. The paper containing this investigation was published in the Philosophical Transactions for 1783; and it is remarkable not only by reason of its giving the first determination of the kind, but on account of the confirmation which the result, though deduced from very insufficient data, has received from subsequent investigations—a circumstance, however, by no means rare in respect of the cosmical speculations of Sir W. HERSCHEL.

After some general considerations respecting the *à priori* probability of the sun's proper motion, Sir W. HERSCHEL, in the paper alluded to, proceeds to describe the phenomena to which it would give rise, namely, a general parallactic motion of the stars, the amount of which, in respect of any star, will depend both on the star's distance from the sun and its situation in the sphere with reference to the point towards which the sun is moving. It is manifest that if we suppose the sun to move in the direction of any assumed point, all the stars which are near enough to our system to be sensibly affected by such a motion, will appear to move towards the point diametrically opposite; and that on one side of the sphere all the right ascensions will appear to increase, while all those on the other side will appear to diminish. He selected seven stars—Sirius, Castor, Procyon, Pollux, Regulus, Arcturus, and α Aquilæ—all of which appeared from comparisons made by Dr. MASKELYNE to have proper motions in right ascension, and two of them—Sirius and Arcturus—also in declination; and finding that the right ascensions of all of them, with the exception of α Aquilæ, appeared to diminish, he assumed the direction of the sun's motion to be from a point "somewhere not far from the 77th degree of right ascension to its opposite 257th degree," the effect of which would be to produce apparent changes of right ascension agreeing with the observed; and he adds, "supposing the sun to ascend at the same time towards some point in the northern hemisphere, for instance towards the constellation of Hercules, then will also the observed change of declination of Sirius and Arcturus be resolved into a single motion of the solar system." In order to test this conclusion he selected twelve stars, quoted by LALANDE from MAYER's table above referred to, the motions of which were assigned both in right ascension and declination; and adding the motions in right ascension of three other stars, he thus obtained twenty-seven changes of position to be accounted for by the hypothesis. By assuming the sun's motion to be directed towards a point "somewhere near λ Herculis," he found that twenty-two of these motions were satisfied, there being only two exceptions in right ascension, and three in declination. The point thus indicated is situated at 257° of right ascension, and 25° of north declination; but he observes that with respect to the changes of declination the point λ Herculis is not, perhaps, the best-selected, as a somewhat more northern situation may agree better with the changes of declination of Arcturus and Sirius, "which capital stars," he thinks, "may be the most proper to lead us in this hypothesis."

In a *Postscript* to his paper, Sir W. HERSCHEL compares the above conclusion with the proper motions of the other stars in MAYER's table, and shows that, out of forty-four stars, the observed motions of thirty-two agree with the hypothesis, while those of the remaining twelve cannot be accounted for by it, and "must therefore be ascribed to a real motion of the stars themselves, or to some still more hidden cause of a still remoter parallax."

It will be remarked that the above result was arrived at without the aid of any calculation whatever, nor does it appear that the precise direction of the apparent motion of any of the stars was ascertained or taken account of. The author considered merely the changes in right ascension and declination, and gave such a direction to the solar motion as would produce corresponding changes in those two directions in the greatest number of instances, without reference to their relative amounts, or attempting to produce an exact coincidence of the hypothetical and apparent directions in any particular case. Nor did he pretend to assign the point towards which the sun's motion is directed with any precision; "it is somewhere near λ Herculis, but may be somewhat more to the north."

In the same year (1783) in which Sir W. HERSCHEL's paper appeared in the Transactions, PREVOST communicated the results of a similar inquiry to the Berlin Academy in a memoir which was published in the Nouveaux Mémoires of that Society for 1781. PREVOST's investigation was also grounded on the proper motions given in MAYER's table. After stating the opinion of MAYER that the observed motions could not be explained on the hypothesis of the motion of the solar system, he remarks, that on examining the table under every point of view, he had come to an opposite conclusion, and found that it did in fact afford indications of such a motion, although the true motions of the stars, or, perhaps, some other cause, occasioned exceptions. He then selects, from MAYER's list, twenty-six stars whose variations of position exceeded 14″ in right ascension or declination, and from a comparison of the whole concludes that the apparent motions indicated by the table would be most nearly represented by supposing the sun's motion to be directed towards that point of the heavens of which the right ascension is 230°, and the declination 25° north,—a conclusion which agrees with that of Sir W. HERSCHEL in respect of declination, but differs from it about 27° in right ascension. The agreement of the individual observations with this result he considered was sufficient to render it probable, first, that the solar system is actually moving towards the point indicated, and, secondly, that at the present time the stars which are the nearest to the sun are Sirius, Procyon and Arcturus; and he thought the observations also afforded grounds for conjecturing that the sun may be describing, *in antecedentia*, an orbit about Arcturus, or at least about a centre of gravity common to those brilliant stars which occupy the quarter of the heavens in which the right ascensions appear to diminish, such as Arcturus, Regulus, Procyon, Sirius*.

* Among the inferences drawn by PREVOST from the hypothesis of the sun's proper motion, the following may be remarked:—supposing comets to be formed of matter existing beyond our system, but projected so as

A third deduction from MAYER's table was made by KLUGEL, in the Berlin Ephemeris for 1789. After giving formulæ for determining from the observed variations in the positions of the stars the direction of the sun's motion in space, he applies them to the proper motions given in the table, and finds the pole towards which the sun's motion is directed to be at the point of which the right ascension is 260° and north declination 27°. This differs from Sir W. HERSCHEL's determination only by 3° of right ascension, and 2° of declination.

Although the general agreement of these three results was calculated to draw the attention of astronomers to the subject, and served, at least, to give a certain plausibility to the hypothesis, no further addition was made to the data of the problem till the publication, in 1790, of Dr. MASKELYNE's table of the proper motions of thirty-six stars. This table, which furnished much more certain data than had previously existed, gave occasion to a second elaborate memoir by Sir WILLIAM HERSCHEL, which appeared in the Philosophical Transactions for 1805.

The mode of proceeding employed by Sir W. HERSCHEL in this memoir merits attention. Having computed from the observed variations of right ascension and declination the apparent direction of the proper motion of each of the stars, he traced on a celestial globe the great circles in which they were contained. On the supposition that the variations in question were parallactic motions caused by the translation of the sun, it was evident that all the great circles containing them would intersect each other in the same two opposite points of the sphere. Now of the intersections thus formed by taking the stars in pairs, he found ten made by six stars of the first magnitude to be contained within a very limited portion of the heavens about the constellation Hercules, while (he remarks) "upon all the remaining surface of the globe there was not the least appearance of any other than a promiscuous situation of intersections, and of these only one was made by arches of principal stars." The six stars which gave the contiguous intersections were Sirius, Arcturus, Capella, Lyra, Aldebaran and Procyon. But six stars combined by pairs give fifteen intersections; of these, therefore, five were rejected, that is to say a third of the whole, as not agreeing with the hypothesis. He then computes, by a trigonometrical calculation, the exact situations of the points of intersection of the ten arches, and (taking the points from which the stars appeared to recede) found them to be all included between 235° and 290° of right ascension, and between 17° and 58° of north declination. He then takes into account the motions of three other large stars, of the second order, and, on combining them with those of the former six, found out of the whole number of new intersections fifteen which agreed with the former in "pointing out the same part of the heavens as a parallactic centre." The positions of these fifteen new points were not calculated, but determined graphically; he conceived, however, they might be depended on as true to one degree of the sphere.

to come within the sphere of the sun's attraction, ought it not (he asks) to happen that more comets will appear in the quarter of the heavens towards which the sun is advancing than the opposite quarter?

The intersections thus found, although lying in the same quarter of the heavens, were not confined within a very narrow space, and in order to obtain a precise result, he proceeds as follows. Confining his attention to the six stars above named, he found the sum of their annual apparent motions in space to be $5''\cdot3537$. Now, assuming the star λ Herculis (as determined in his first paper) to be the point towards which the sun is moving, he computes the angle included between the great circle of the sphere which passes through this point and the star, and the great circle in which the star's motion takes place according to the comparison of the catalogues; he then multiplies the apparent quantity of the annual proper motion of the star by the cosine and sine respectively of this angle, whereby the apparent proper motion is resolved into two parts,—one in the direction in which the star would appear to move in consequence of the hypothetical motion of the sun, and the other at right angles to that direction. The first of these may be ascribed to the motion of the sun; the second must be regarded as due to the true proper motion of the star. Adding, therefore, into one sum the former of these resolved parts for each of the six stars, and deducting the sum from the sum of the observed annual motions, the latter sum was reduced from $5''\cdot3537$ to $2''\cdot2249$. By assuming another point in the same constellation as the apex of the sun's motion, the sum of the annual proper motions of the six stars, in the direction perpendicular to that resulting from the hypothesis, was further reduced to $1''\cdot4594$; and after some other trials, he ultimately fixed upon the point (near 34 Herculis) whose right ascension was $245° 52' 30''$, and north declination $49° 38'$, by which the sum of the true annual proper motions of the six stars was reduced to $0''\cdot9559$. He concluded that this point must be very near the truth, inasmuch as "the alteration of a few minutes in right ascension or north polar distance, either way, would immediately increase the required real motion of our stars."

This determination of the position of the solar apex differs from that which was given in Sir W. HERSCHEL's former paper by about $11°$ of right ascension, and $24° 38'$ of declination. It rests, however, on the proper motions of only six stars, and, therefore, notwithstanding the greater probable accuracy of the observations, and the more elaborate process of calculation by which it was arrived at, it is probably not of greater intrinsic value than the first. The principle on which it is based, namely, the supposition that the sum of the true proper motions of the stars is a minimum, and consequently that the direction to be assigned to the sun's motion must be that which will account for the greatest amount possible of the observed motions, was objected to by BURKHARDT, on the ground that there is no more reason for supposing the sum of the true proper motions to be a minimum than a maximum, excepting on the hypothesis that the stars are more inclined to rest than to motion. But this objection seems to imply some misapprehension of the problem under consideration. No hypothesis respecting the disposition of the stars to rest or to motion is involved. The apparent proper motions are the results of the comparison of the catalogues, and the question proposed by Sir W. HERSCHEL was simply to determine the point towards which the sun must be supposed to move, in order that, after deducting the parallactic effect, the amount of the residual motions might be the least possible. BURKHARDT's memoir was published in the Connaissance des Temps for 1809. It contains formulæ for the solution of the problem, with their application to several of the stars in MASKELYNE's catalogue; but he found little accordance among the results, and concluded that we are not yet in possession of a sufficient number of facts to decide on the direction of the sun's motion.

BIOT, in the Additions to his Astronomie Physique, also considered the question of the translation of the planetary system, and gave formulæ for determining the right ascension and declination of the solar apex. He computed the intersections of the great circles containing the arcs described by eight stars, viz. Aldebaran, Capella, Sirius, Procyon, Pollux, Arcturus, α Lyræ, and α Aquilæ, the proper motions of which were given by ZACH from a comparison of BRADLEY's places with those of MASKELYNE, and also of the catalogues of MAYER and PIAZZI. If the apparent motions depended solely on the displacement of our system, the intersections would, of course, all be found at the same two points of the sphere; but he found the discrepancies to be so great that he considered them to be irreconcileable with the supposition of their dependence on any systematic motion or common cause. "The examination," he remarks, "of all these irregularities shows that the stellar motions hitherto observed are not subject to any law, and that it would be in vain to attempt to reconcile them by supposing them all to be directed towards the same pole. Hence it becomes infinitely probable that such of these motions as are well determined are due, in part, to a real displacement of the stars themselves, and not to that of our system. With respect to those whose proper motions are less certain, not only does their want of precision prevent us from concluding from them the direction of the motion of our own system, but their comparison does not even afford any indication which can lead to the inference that it is in motion at all."

In the 12th section of his Fundamenta Astronomiæ, BESSEL has given an elaborate investigation of this question, founded on a much larger number of proper motions (and probably better determined) than had previously been brought to bear on the inquiry. On comparing the catalogue deduced from BRADLEY's observations with that of PIAZZI, he found seventy-one stars having a proper motion of not less than $0''\cdot5$ annually in the arc of a great circle, and computed the positions of the great circles in which the apparent motions are contained. But even from this large number he obtained no satisfactory or conclusive result. The investigation, he remarks, did not confirm HERSCHEL's supposition of the sun's motion towards the constellation Hercules, since many points on the sphere, very remote from each other, and even diametrically opposite, may be assigned which are situated in the direction of the motion of many stars; but whatever point may be taken, there will always be found so many proper motions evidently receding from it, that no sufficient reason will remain for preferring one point to another. And he concluded that a very long time must elapse before any proficiency would be made in the theory of the proper motions of the stars.

The opinion of Bessel, now quoted, appears to be that which, until lately at least, has been generally entertained by astronomers; but on attentively considering the nature of the question, it will soon be seen that none of the methods of investigation yet alluded to can be considered as capable of leading to an entirely satisfactory conclusion. They are all founded essentially on the principle of determining the apex of the sun's motion from the apparent motions of single pairs of stars; and, with the exception of Bessel's, all the results which had been given were deduced from a very small number of proper motions. Now, it is a very improbable supposition that the stars are subject to no variations but such as depend on the motion of the sun. We must suppose them to have true proper motions, producing apparent effects at least equal in amount to those which are supposed to be produced by the sun's displacement. Assuming, then, that the stars themselves are in motion as well as the sun, and that they move in all directions, the appearances will necessarily be of a very complicated nature. The proper motions of some stars will conspire with that of the sun, and increase the apparent change of position. In other cases they will be contrary to that of the sun, and the apparent effect will be that which is due to the difference of two real motions. In general the directions of the true and parallactic motions will be inclined to each other; but in all cases the difference given by the comparison of the catalogues will be compounded of the effect of the real motion of the star and the effect of the sun's displacement. Hence it is manifest that the fact of proper motions being observed to take place in all directions, is in no way inconsistent or incompatible with an apparent general drifting of the stars towards one particular region; and the problem to be solved is to separate, if possible, the general effect produced by the sun's displacement from the complicated effects caused by the motions of stars in every direction with which it is entangled and mixed up. Now it is easy to see that a question of this kind cannot be solved by taking account of only a small number of proper motions. A very considerable number must be employed; and, indeed, in order that the solution may be satisfactory, regard must be had to every star without exception of which the proper motion has been determined with sufficient certainty. It is also necessary that the investigation be conducted in such a manner that every observed displacement shall contribute, according to its weight, to the general result; and the probable error of the result must likewise be determined in order that the relative probabilities of results obtained from different hypotheses respecting the direction of the sun's motion, may be submitted to exact comparison. In this manner it will be seen whether, as Bessel and others inferred from particular cases, numerous points may be assumed towards any one of which the sun may be supposed with equal probability to be advancing, or whether there is so great a preponderance of observed motions towards one particular region as to warrant the assumption of a systematic origin, or make evident the operation of a common cause. This mode of considering the question was first adopted by ARGELANDER, in the memoir alluded to at the commencement of this paper.

ARGELANDER'S investigation, as already stated, is founded on the proper motions of 390 stars, determined by a comparison of their mean places in 1755, according to BESSEL'S reduction of BRADLEY'S observations, with their mean places in 1830, as given in his own catalogue*, deduced from observations made by himself at Abo, the interval between the epochs being seventy-five years. In this investigation every star was included which appeared, on comparison of the two catalogues, to have undergone a change of position to the extent of $7''\cdot5$, or to have an annual proper motion amounting to $0''\cdot1$ in space. By reason of the excellence of both catalogues, the long interval between their respective epochs, and the very considerable number of stars employed, the result must be considered as by far the most satisfactory that had yet been given.

The method of calculation employed by ARGELANDER may be described generally as follows:—1. From the variation in right ascension and declination given by the comparison of the catalogues, the angle (ψ) is computed which the apparent path described by the star makes with the circle of declination. 2. A point (Q) is assumed as the apex of the sun's motion, and the direction in which the star would appear to move (if it had no real motion of its own) in consequence of the motion of the sun, is computed from the position of the star and the assumed position of the point Q, and expressed in terms of the angle (ψ') which it makes with the declination circle. 3. The trigonometrical value of ψ' is differentiated on the supposition that the right ascension (A), and declination (D) of the point Q are variable quantities, and in the resulting expression the numerical value of the difference of the angles ψ and ψ' is substituted for $d\psi'$, by which means an equation is obtained in which there are only two undetermined quantities, viz. dA and dD. Each star furnishes a similar equation; and as the effect of the real motion of the sun on the apparent displacement of any star is proportional to the sine of the star's distance from the apex of the sun's motion, each equation is multiplied by the corresponding sine of this distance, whereby they are all reduced to the same degree of precision. 4. The equations are then solved by the method of least squares, and the resulting values of dA and dD applied as corrections to the assumed values of A and D, which determine the situation of the point Q. With the corrected values of A and D thus obtained, the angles ψ' may be recomputed, and after one or two repetitions of the same process, values of A and D will be obtained giving the position of Q which most nearly represents the whole of the observations.

The effect of the sun's displacement on the apparent proper motion of a star is inversely proportional to the distance of the star from the sun; but this distance

* DLX Stellarum fixarum positiones mediæ ineunte anno 1830. Ex observationibus Aboæ habitis deduxit . . . . . . Fr. ARGELANDER. Helsingforsiæ, 1835. 4to.

being entirely unknown, no account can be taken of it excepting upon some assumption more or less arbitrary. ARGELANDER assumes as a probable hypothesis, that those stars which have the largest proper motions are the nearest to our system, and introduces the condition of relative proximity by dividing the stars upon which his calculation was made into three classes, and giving different weights to the equations belonging to the different classes, all the stars in the same class being assumed to be at the same mean distance. The first class contained twenty-one stars, having proper motions exceeding one second of arc annually; the second contained fifty stars whose annual proper motions are between $1''\cdot0$ and $0''\cdot5$; and the third the remaining 319 stars, the annual proper motions of which were included between $0''\cdot5$ and $0''\cdot1$. The partial results deduced from each class presented a nearer agreement than was, perhaps, to be anticipated from the nature of the question. In giving an account of his memoir in No. 363 of the Astronomische Nachrichten, ARGELANDER corrects some errors of calculation which had escaped detection in the original paper, and states the most probable values (with their probable errors) of the right ascension and declination of Q, as resulting from the combination of the whole of the equations of condition, to be as follows:—

$$A=259^\circ\ 47'\cdot6\pm3^\circ\ 18'\cdot6,\quad D=+32^\circ\ 29'\cdot5\pm2^\circ\ 13'\cdot5,$$

for 1792·5 (the mean epoch of the catalogues), or

$$A=259^\circ\ 51'\cdot8,\quad D=+32^\circ\ 29'\cdot1,$$

when reduced to the beginning of 1800.

This result differs very considerably from that which was obtained by Sir W. HERSCHEL in his paper of 1805, viz. $A=245^\circ\ 52'\ 30''$, $D=+49^\circ\ 38'$, but approximates nearly to the determination in the paper of 1783; the difference from the latter being less than 3° in right ascension, and about $7\frac{1}{2}^\circ$ in declination.

In order to give an idea of the probable accuracy of this result, ARGELANDER deduces the following conclusions. If with the point Q thus found as a centre, and a radius containing $3^\circ\ 45'\cdot7$, a circle be described on the sphere, the wager is 1 to 1 that the sun's motion is directed to some point within this circle; 14 to 3 that it is directed to some point within a circle having the same centre and a radius of $7^\circ\ 31'\cdot4$; 89 to 4 that it is directed to a point within a circle having the same centre and a radius of $11^\circ\ 17'\cdot1$; 142 to 1 that the point will be within a circle having the same centre and a radius containing $15^\circ\ 2'\cdot8$; and if we increase the radius to $18^\circ\ 48'\cdot5$, the wager will be more than 1341 to 1 that the point Q will lie somewhere within that circle.

In No. 398 of the Astronomische Nachrichten, ARGELANDER returns to the subject a third time, and gives another determination of the direction of the solar motion, calculated by LUNDAHL from a different set of stars. The Abo catalogue does not contain the whole of BRADLEY's stars given in the Fundamenta Astronomiæ, and on comparing the latter work with POND's catalogue of 1112 stars (reduced to the beginning of 1830), LUNDAHL found 147 not included in ARGELANDER's investigation, whose proper motions appeared to be not less than $0''\cdot09$ of space annually. Having first recomputed the places of those stars with a more exact value of the precession, and applied to POND's observations the correction necessary to render them strictly comparable with those of ARGELANDER, LUNDAHL assumed the apex of the sun's motion to be situated at the point indicated by ARGELANDER's investigation, and calculated the value of ψ' for each star on this assumption. Comparing the directions thus obtained with those of the apparent motions, and forming the equations of condition according to the method of ARGELANDER, the resulting values of the right ascension and declination of Q were found to be

$$A=252^\circ\ 24'\cdot4\pm5^\circ\ 25'\cdot3\ ;\quad D=+14^\circ\ 26'\cdot1\pm4^\circ\ 29'\cdot3.$$

This result differs from that of ARGELANDER more than 8° in right ascension, and about 17° in declination; and the corrections of the assumed values are far beyond the limits of the probable errors assigned by ARGELANDER. By reason, however, of its smaller weight, it does not, when combined with the former determinations, materially alter the probable situation of the point Q. Combining it with the results of each of his three classes, with due regard to their relative weights, ARGELANDER gives the following values of the coordinates of Q as the most probable result of the whole of the observations:—

$$A=257^\circ\ 49'\cdot7\pm2^\circ\ 49'\cdot2\ ;\quad D=+28^\circ\ 49'\cdot7\pm1^\circ\ 59'\cdot8.$$

A still more recent attempt to assign the position of the apex of the sun's proper motion has been made by OTTO STRUVE, the results of which are given in a paper published in the Petersburg Memoirs (tome 5) for 1842. This investigation is grounded on the proper motions of about 400 stars, as determined by a comparison of their mean places in 1755, according to BESSEL's catalogue, with their positions in 1825 deduced from observations made at the Dorpat Observatory. Of the whole number of stars employed, only 134 are included among those from which ARGELANDER's result was deduced, so that about 260 additional proper motions are brought to bear on the hypothesis. The mode of investigation is different in several respects from that which has been described. Assuming the direction of the sun's proper motion to be determined, the proper motions indicated by the comparison of the catalogues are manifestly functions of the constant of precession used in reducing BRADLEY's places to 1825, and of the quantity of the solar motion. From the equations of condition furnished by the observed variations of right ascension and declination, he determines the precession and the angular motion of the sun (which, as seen from the mean distance of stars of the first magnitude, he finds to be $0''\cdot339$ in a year, with a probable error of $0''\cdot025$), and having substituted these values in the equations of condition, he employs the residual errors in forming a new system of equations which serve to determine dA and dD, the corrections in right ascension and declination of the assumed direction of the solar motion. The directions of the apparent proper motions were not computed, but the variations in right ascension and declination expressed separately in terms of the assumed values of A and D and the star's place, so that each star furnishes two independent equations. For determining the relative weights of the equations, he adopts the hypothesis that the distances of the stars are inversely as their apparent magnitudes; and, dividing all the stars from the first to the seventh magnitude inclusive into twelve classes, he assumes, on grounds given by the elder STRUVE, in the Introduction to his catalogue of double stars, the mean distance of those in the first class $=1$, of those in the second $=1{\cdot}71$, and so on to the twelfth class, or seventh magnitude, for which the mean distance becomes 11·34. Weights depending on those distances were assigned to the equations furnished by the stars in each of the twelve classes, and the values of dA and dD deduced. The result gave the position of Q, for 1790, as follows:—

$$A=261^\circ\ 21'{\cdot}8\pm4^\circ\ 49'{\cdot}9;\quad D=+37^\circ\ 36'{\cdot}0\pm4^\circ\ 11'{\cdot}8.$$

For the sake of comparison, I here subjoin the results of the several investigations of ARGELANDER, LUNDAHL, and OTTO STRUVE.

Position of the apex of the sun's proper motion for 1792·5,

		A=	D=	
ARGELANDER	I.	256° 25′·1±12° 21′·3	+38° 37′·2±9° 21′·4	(21 stars).
ARGELANDER	II.	255° 9′·7± 8° 34′·0	+38° 34′·3±5° 55′·6	(50 stars).
ARGELANDER	III.	261° 10′·7± 3° 48′·9	+30° 58′·1±2° 31′·4	(319 stars).
LUNDAHL	IV.	252° 24′·4± 5° 25′·3	+14° 26′·1±4° 29′·3	(147 stars).
O. STRUVE	V.	261° 23′·1± 4° 49′·9	+37° 35′·7±4° 11′·8	(392 stars).

From these five determinations O. STRUVE deduced the following mean result:—

$$A=\ 259^\circ\ 9'{\cdot}4,\ \text{with a probable error}=2^\circ\ 57'{\cdot}5.$$
$$D=+\ 34^\circ\ 36'{\cdot}5,\ \text{with a probable error}=3^\circ\ 24'{\cdot}5.$$

The proper motions on which the following investigation is grounded are deduced from a comparison of the mean positions of eighty-one stars in the southern hemisphere, as observed by Mr. JOHNSON and the late Professor HENDERSON, with the positions assigned to them in the catalogues of LACAILLE and BRADLEY. Every star, without exception, has been included, which, from the differences of right ascension and declination given in the two recent catalogues, appears to have a proper motion amounting to $0''{\cdot}1$ in space, or upwards, annually.

Mr. JOHNSON's catalogue, which was published in 1835, gives the mean positions of 606 stars, observed by him at St. Helena, and reduced to the beginning of 1830. Of these stars a considerable number are contained in LACAILLE's catalogue, originally published in the Astronomiæ Fundamenta (1757), and recently by Mr. BAILY in vol. v. of the Memoirs of the Royal Astronomical Society. The epoch of LACAILLE's catalogue is 1750, so that the interval is eighty years. In order to compare the two catalogues, Mr. JOHNSON reduced the positions given by LACAILLE to the epoch 1830, by applying the precession due to the middle epoch, 1790; and on examining the proper motions of the stars so compared, I have found fifty-six which appear to have changed their positions 8″·0 or upwards in the arc of a great circle, or to have an annual proper motion of not less than 0″·1. To these have been added five others whose annual proper motions appear to be somewhat less than 0″·1, according to Mr. Johnson's observations, but to amount to that quantity according to Henderson's determination compared with that of Lacaille. The whole number of stars, therefore, included in the present inquiry, whose proper motions are deduced from the comparison of the observations of Mr. Johnson with those of Lacaille, is sixty-one.

Henderson's catalogue contains the mean right ascensions and declinations of 172 of the principal southern stars, being part of a very much larger number observed by him during his short residence at the Cape in 1830 and 1831, while in charge of the Government Observatory established in that colony, the reduction of which unfortunately he did not live to complete. The declinations were published in 1838, in vol. x. of the Memoirs of the Royal Astronomical Society, and the right ascensions in vol. xv. of the same series, which appeared only last year (1846). In the latter volume he gives a list of fifty-two stars which appear to have a proper motion of not less than 0″·1 annually, deduced in thirty-six cases from a comparison of his own observations with those of Lacaille, and in the remaining sixteen cases with those of Bradley. Henderson's mean places are for the beginning of 1833; so that the interval is eighty-three years in the case of comparison with Lacaille, and seventy-eight years in the case of comparison with Bradley. The whole of the thirty-six stars compared with Lacaille, are contained in Mr. Johnson's catalogue, but there are four of them in respect of which the comparison is not given by Mr. Johnson, and which, therefore, are not included among the sixty-one above referred to. Henderson's catalogue, therefore, gives twenty additional stars, so that on the whole the number taken into account is eighty-one; namely, sixty-five whose proper motions depend on Lacaille's observations, and sixteen on the observations of Bradley.

For the purpose of deducing the direction of the apparent proper motion from the observed variations of right ascension and declination, as well as for determining the hypothetical direction, every star has been referred to its mean position for 1790. In the case of Mr. Johnson's stars, this reduction has been made by taking the mean right ascension and the mean declination of the two compared catalogues. In respect of the stars in Henderson's catalogue, those which are compared with Lacaille's places were first reduced to 1830 by applying the precession given in the catalogue, and the mean then taken between these reduced places and the places of Lacaille; and in the case of those compared with Bradley's catalogue, the mean of the two catalogues gave the positions for 1794, from which they were reduced to 1790, by applying the precession for that epoch. With respect to the stars common to the catalogues of Mr. Johnson and Henderson, the annual variation both in right ascension and declination was first deduced from the comparison of each catalogue with LACAILLE's places, and the mean of the two comparisons then taken and made use of in the subsequent calculations.

Having thus obtained the places of the stars for the mean epoch 1790, and the annual variations in right ascension and declination being found by dividing the differences of the catalogues (expressed in seconds of arc) by the number of years in the interval, the angle ψ, which the apparent path of the star makes with the circle of declination, was computed. This angle determines the position of the great circle of the sphere in which the apparent motion takes place.

The next step in the process is to compute the angle ψ', or the direction in which the star would appear to move in consequence of the translation of the sun towards an assumed point Q. With respect to this point, or apex, the position which may be regarded as the most probable is, perhaps, that which was deduced by OTTO STRUVE from the five determinations above given; but the object here was not to choose the point which has the greatest probability in its favour, but that which appeared the most likely to satisfy the present observations. Now, on examining the apparent motions of the stars under consideration, it was easy to see that the apex must have a considerably greater declination than that which was assigned to it by LUNDAHL. OTTO STRUVE's result, on the other hand, which differs from the mean in the opposite direction, appeared to me to be less trustworthy from the manner in which it was deduced. I therefore assumed, as the apex of the solar motion, the point determined by ARGELANDER, though, as it turns out, the motions in declination would have been somewhat better satisfied by assuming the mean of all the results as given by OTTO STRUVE. Its position for 1790 (the mean epoch of the catalogues) is

$$A=259^\circ\ 46'\cdot 2,\quad D=+32^\circ\ 29'\cdot 6^*.$$

From these values of A and D the angle ψ' was computed for each of the eighty-one stars separately. The results, as well as the values of ψ, and the differences $\psi-\psi'$, will be found in a table hereto subjoined.

Before proceeding to state the results obtained from the equations of condition, it will be worth while to examine the presumptions for or against the hypothesis deducible from the comparison of the directions of the apparent proper motions of the different stars, and the directions of the parallactic motions which would result from the motion of the sun towards the assumed point Q.

First, with respect to the observed variations of right ascension: if we conceive the celestial sphere to be divided into two hemispheres by a great circle passing through

* This is the position according to the values of the right ascension and declination given by ARGELANDER in No. 363 of the Astronomische Nachrichten, but in a subsequent number of the same work (No. 398) it is stated that an error of calculation had been committed the correction of which would have given the position of Q for 1792·5 as follows: Æ$=260^\circ\ 51'$, Dec. $=+31^\circ\ 17'$. The correction was not observed till after the values of ψ' had been calculated for all the stars; but for the present purpose the difference is manifestly of no importance.

Q and perpendicular to the equator, the effect of the sun's motion towards Q would be to increase the right ascensions of all the stars in one of the hemispheres, and to diminish the right ascensions of all those in the other. Now, out of the whole of the stars compared, thirty-five are situated in the hemisphere in which the right ascensions should increase according to the hypothesis, and of these there are twenty-three whose right ascensions have actually increased, and twelve whose right ascensions have diminished; that is to say, there are twenty-three instances favourable to the hypothesis, and twelve unfavourable. In the other hemisphere there are forty-six stars; and of these the number whose right ascensions have diminished, agreeably to the hypothesis, is thirty-nine, and the number whose right ascensions have increased, contrary to the hypothesis, is seven. Hence it appears that in respect of the eighty-one stars included in the inquiry the observed motions in right ascension are favourable to the hypothesis in sixty-two instances, and unfavourable in nineteen. Allowing the same weight to each instance, the wager is therefore sixty-two to nineteen, or somewhat more than three to one in favour of the hypothesis of a common tendency towards a determinate region.

Secondly, with respect to the observed variations of declination. In all cases in which the angle ψ' is less than 90°, or greater than 270°, the effect of the sun's motion in the direction of the assumed point Q, is to bring the apparent place of the star towards the north, so that its declination (which is south in all cases) should appear, on comparison of the catalogues, to have diminished; and in all cases in which ψ' is greater than 90° and less than 270°, the variation in the place of the star should be towards the south, and the declination should increase. Now, in the first case there are fifteen stars, and of these ten have advanced towards the north, agreeably to the hypothesis, and five towards the south, contrary to the hypothesis. In the second case there are sixty-six stars; and of these the observed motion of fifty-three is towards the south, agreeing with the hypothesis, and of thirteen towards the north, contrary to the hypothesis. On the whole, therefore, in respect of declination, there are sixty-three instances favourable to the hypothesis and eighteen unfavourable; and the wager is seven to two in favour of the hypothesis. It may be added that there are only three stars out of the whole number (Nos. 18, 34 and 36 in the subjoined table) whose observed proper motions are contrary to the hypothesis both in right ascension and declination.

Another inference may be drawn from the comparison of the angles ψ and ψ'. If the observed changes of position were wholly independent of the sun, all directions would be equally probable, and it would be an even wager that the difference between ψ and ψ' would be less or greater than 90° in any case, since all possible values of that difference lie between 0 and 180°. But there are only ten instances, out of eighty-one, in which the difference exceeds 90°.

From this general agreement of the hypothetical and observed motions, a strong presumption is raised in favour of the hypothesis; for it can scarcely be supposed that the agreement would hold good in so great a majority of instances if it were purely the effect of chance. But a much more certain conclusion will be arrived at from the combination of the whole of the observations by the method of least squares.

The method of forming the equations of condition has already been explained generally, and the formulæ for computation will be given in subsequent paragraphs; but as some of the stars are more favourably circumstanced than others for determining the question at issue, it becomes necessary, before proceeding with the solution, to assign weights to the equations, in order to reduce them all to the same precision, and obtain the most probable values of the corrections to be applied to the assumed position of the solar apex. For this purpose some special considerations are required.

Admitting the hypothesis of the sun's motion, it can hardly be supposed that any star is absolutely at rest. The apparent motion of a star, therefore, as it is made known to us by a comparison of observations, is the effect of the combined motion of the sun and the star. Now, with respect to the true proper motion, we are in ignorance of all the circumstances by which its apparent or visible effect is modified. We know nothing whatever respecting the magnitude or nature of the orbit described by the star, or the absolute velocity with which it moves. Hence it is necessary to assume that all the stars move with the same absolute velocity, in which case (putting the sun's motion out of consideration) the apparent velocity will be inversely as the distance. But we are equally ignorant of the relative distances, and are therefore reduced to the necessity either of disregarding the distance altogether, or of making some precarious assumption respecting it,—for instance, that the distances of the different stars are inversely proportional to their magnitudes (as in the method of Otto Struve), or inversely as the quantities of the apparent proper motions. In the present inquiry no greater probable accuracy could be obtained by the adoption of either of these assumptions, and consequently errors to which differences of distance as well as of absolute velocity give rise, are regarded as constant. The only remaining circumstance by which the apparent effect of the true proper motion is modified is its direction; and as there is no *à priori* reason for assuming that a star is more likely to move in one direction than another, all directions must be regarded as equally probable. The conclusion, therefore, is, that in respect of the true proper motions *inter se*, we have no sufficient grounds for making any distinction as to the relative precision of the results given by different stars, so that the errors arising from this cause must be treated as accidental errors of observation, and all the equations be allowed to have the same weight.

With respect to the part of the apparent motion depending on the displacement of the sun, the case is different, inasmuch as the parallactic effect depends not only on the distance of the star, but also on its situation with respect to the apex of the sun's motion. The effect of the sun's motion on the observed position of a star (as will be shown more particularly further on) is directly as the sine of the star's distance from the point Q towards which the sun is moving; and hence, in order that all the equations may have the same weight, each must be multiplied by the sine of that distance. In other words, if $\varepsilon(\Psi)$ denote the probable error in the observed direction of the proper motion, or the probable value of $\psi-\psi'$, for a star at the distance of 90° from the point Q, then $\varepsilon(\Psi)$ will also be the probable value of $(\psi-\psi')\sin\chi$ for a star whose distance from Q is measured by the angle χ. Hence it follows that every value of $\psi-\psi'$ must be multiplied by $\sin\chi$.

The eighty-one equations of condition are given in an appended table. They are of the following form,

$$0=+a dA+b dD-n,$$

where a, b, and n are numbers deduced from the data, and dA, dD the quantities to be determined from the equations and applied as corrections to the assumed values of A and D. Forming the squares and products of these numbers, and adopting, according to the usual notation, (aa) to denote the sum of the squares of the coefficients of dA, (bb) the sum of the squares of b, (ab) the sum of the products of a and b, and so on, the following values are found:—

$$(nn)=178660{\cdot}4,\ (aa)=38{\cdot}5423,\quad (bb)=26{\cdot}7425,$$
$$(ab)=-5{\cdot}6852,\ (an)=-129{\cdot}462,\ (bn)=-105{\cdot}693,$$

and consequently the two following equations for determining dA and dD, viz.

$$0=+38{\cdot}5423dA-\ 5{\cdot}6852dD-129{\cdot}462,$$
$$0=-\ 5{\cdot}6852dA+26{\cdot}7425dD-105{\cdot}693,$$

the solution of which gives

$$dA=+4^{\circ}{\cdot}070 \text{ with the weight } 37{\cdot}333,$$
$$dD=+4^{\circ}{\cdot}817 \text{ with the weight } 25{\cdot}904.$$

On computing $\varepsilon(\Psi)$, or the probable value of $(\psi-\psi')\sin\chi$, from the appropriate formula of the method of least squares, we find $\varepsilon(\Psi)=31^{\circ}{\cdot}98$; whence, and from the above weights, the probable errors of dA and dD are respectively $5^{\circ}{\cdot}234$ and $6^{\circ}{\cdot}285$. The result, therefore, of the whole calculation from the assumed values of A and D, namely $A=259^{\circ}\ 46'{\cdot}2$, $D=+32^{\circ}\ 29'{\cdot}6$, gives the following values of A and D for the position of the point Q for the beginning of 1790,

$$A=263^{\circ}\ 50'{\cdot}4\pm5^{\circ}\ 14'{\cdot}0;\ D=+37^{\circ}\ 18'{\cdot}6\pm6^{\circ}\ 17'{\cdot}1.$$

This result presents a very remarkable agreement with that obtained by Otto Struve from the Dorpat observations; the values of dA and dD are, however, somewhat greater than the probable errors of the hypothesis, according to the determination of Argelander.

Following out the principle of the method, the next step would be to recompute the angles ψ', and the equations of condition, with the values of A and D now found, so as to obtain a result having a smaller probable error; but, in the present case, the labour attending a new calculation (by no means inconsiderable) is altogether unnecessary, as will appear from the following considerations.

In the first place it is to be remarked, that the result has been deduced from all the stars whose annual proper motions were found to be not less than $0''\cdot1$, without any selection or rejection on account of obvious discrepancies. But it is manifest that if there is observed a general tendency to motion in a determinate direction, while in one or two instances the motion is in a nearly opposite direction, the presumption will be that in such exceptional cases the disagreement arises from the circumstance that the parallactic motion is masked and concealed by the relatively greater proper motion of the star. In the second place, a few of the stars under consideration are very unfavourably situated for correct determination of the right ascension according to the method practised by LACAILLE, and also for correct comparison by reason of the uncertainty of the computed precession; and in such cases a disagreement with the general result naturally gives rise to a suspicion of error in the determination. Now there are two stars, β and γ^1 Octantis (Nos. 15 and 18 in the table) which are in those circumstances. The difference between the observed and hypothetical directions of their motion, or $\psi-\psi'$, is $158^\circ\ 58'\cdot0$ in the case of the first, and $179^\circ\ 13'\cdot0$ in that of the second, showing in the latter case an apparent motion almost directly opposite to the parallactic motion due to the hypothesis. Both stars are also situated within 8° of the pole, so that the determination of their right ascensions by LACAILLE's method of equal altitudes must be liable to considerable uncertainty. For these reasons it may be concluded that the probable accuracy of the result will be increased by rejecting those two stars from the calculation.

Omitting, therefore, the two equations 15 and 18, the sums of the squares and products of the numerical quantities in the remaining seventy-nine are as follows:

$$(nn)=137120\cdot9,\ (aa)=37\cdot1490,\quad (bb)=26\cdot7010$$
$$(ab)=-5\cdot9259,\ (an)=+110\cdot849,\ (bn)=-64\cdot282;$$

which give the following solution,

$$dA=-\ 2^\circ\ 41'\cdot8\pm4^\circ\ 44'\cdot9,$$
$$dD=+\ 1^\circ\ 48'\cdot5\pm5^\circ\ 36'\cdot0$$
$$\varepsilon(\Psi)=\ 28^\circ\ 25'\cdot2.$$

Here the corrections, both in right ascension and declination, are considerably diminished, and it will be remarked that the former has changed sign and become subtractive instead of additive. The probable errors are also considerably reduced, and both corrections are now within the limits of the probable errors of ARGELANDER's determination.

On applying the above corrections to the assumed values of A and D, we obtain, as the result of the calculation from seventy-nine stars,

$$A=257^\circ\ 4'\cdot4\pm4^\circ\ 44'\cdot9,\ D=+34^\circ\ 18'\cdot1\pm5^\circ\ 36'\cdot0.$$

Another omission will diminish the corrections and probable errors still further. One of the stars in HENDERSON's catalogue (η Ophiuchi, No. 80 in the subjoined table) appears to move in a direction nearly opposite to that of the parallactic motion resulting from the assumed hypothesis, the difference of the angles ψ and ψ' being in this case 173° 4′·9. If we reject this star also, on account of the great probability there is that the apparent motion is here due to the excess of the true proper motion of the star above the parallactic motion, we shall have from the remaining seventy-eight stars,

$$(nn)=120332{\cdot}5,\ (aa)=36{\cdot}4473,\ (bb)=26{\cdot}6875,$$
$$(ab)=-5{\cdot}8285,\ (an)=+2{\cdot}308,\ (bn)=-49{\cdot}213,$$

whence the following results,

$$dA=+0^\circ\ 14'{\cdot}4\pm4^\circ\ 31'{\cdot}4,$$
$$dD=+1^\circ\ 53'{\cdot}8\pm5^\circ\ 17'{\cdot}2,$$
$$\varepsilon(\Psi)=\ 26^\circ\ 49'{\cdot}8.$$

Applying these corrections to the assumed values of A and D, the position of the point Q, for 1790, is found as follows :—

$$A=260^\circ\ 0'{\cdot}6\pm4^\circ\ 31'{\cdot}4,\quad D=+34^\circ\ 23'{\cdot}4\pm5^\circ\ 17'{\cdot}2.$$

These values of A and D are almost identical with those which were deduced by OTTO STRUVE from the combination of his own result with those of ARGELANDER and LUNDAHL.

From the near agreement of these results with the hypothesis, it is manifest that it would be an entirely useless labour to recompute values of ψ' from slightly altered values of the right ascension and declination of Q, the corrections of the assumed values being so far within the limits of the probable errors. So close a coincidence, whether accidental or otherwise, is not a little remarkable. In fact the southern stars would seem to accord with the hypothesis even better than those in the other hemisphere ; for the mean value of $(\psi-\psi')\sin\chi$, or $\varepsilon(\Psi)$, in respect of the whole of the stars, is less than the mean found by ARGELANDER from his second and third classes ; and if we leave out the two stars above mentioned near the pole, it is less even than that given by his first class, the values for his three classes being respectively 31° 31′·0, 32° 36′·6, 35° 41′·6.

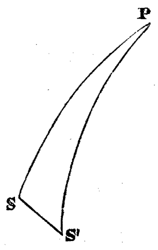
It is difficult to form a satisfactory estimate of the probable accuracy of the result of this calculation, as compared with the results of ARGELANDER and OTTO STRUVE. The number of stars, though not large, might perhaps be regarded as sufficient to render the result worthy of confidence if the proper motions in right ascension and declination indicated by the comparison of the catalogues could be safely relied on ; but, unfortunately, in the present case, the probable errors of observation are hardly susceptible of exact appreciation, and the result is of course affected by the uncertainty of the data. With respect to the two recent catalogues, there is, indeed, no difficulty, inasmuch as the probable errors can be estimated with sufficient precision. Mr. JOHNSON considers the probable error in right ascension of a position given by the mean of five observations to be $0^s{\cdot}034\times\sec\delta$ in time (δ being the declination of the star), or $0''\cdot51\times\sec\delta$ in arc. Taking, as a mean value, $\delta=45°$, this gives the probable error in right ascension $=0''\cdot72$. He also states the probable error in declination, from five observations, to be $0''\cdot35$, exclusive of the error occasioned by uncertainty of refraction. Assuming the average error in declination from all causes to be the double of this, or $0''\cdot70$, we shall have the probable error in the place of the star, in the arc of a great circle, $=\sqrt{\{0''\cdot72^2+0''\cdot70^2\}}=1''\cdot0$. In the case of HENDERSON's catalogue, the probable errors may be regarded as still smaller, owing to the superiority of the instruments of the Cape Observatory. But with respect to LACAILLE's observations, there is considerable uncertainty. His right ascensions were not determined, as in modern practice, by means of a transit instrument, but by the method of equal altitudes, with a 3-foot quadrant; and it is not certain whether the clock, on the accurate performance of which during the interval of the two observations of altitude the result mainly depends, was compensated for temperature. The declinations were observed with a 6-foot sector and a 6-foot sextant; and it is to be remembered that some of the most important elements of reduction—the aberration, nutation, refraction—were then imperfectly known. On the other hand, LACAILLE's well-known skill as an observer, the care he bestowed on the catalogue in the Fundamenta Astronomiæ, and the repeated examinations it has undergone by DELAMBRE and others, may be considered as rendering his positions trustworthy within limits which warrant their application to the purpose in hand. DELAMBRE, who had made extensive comparisons of LACAILLE's observations, estimated the probable error of one of his positions as double the probable error of one of BRADLEY's. But the probable error in declination of a star observed by BRADLEY is estimated by BESSEL at $0''\cdot7$; and the probable error in right ascension of an equatorial star, or, generally, the probable error of $\alpha\times\cos\delta$, is nearly the same as the probable error in declination; whence the probable error in the position of a star on the arc of a great circle may be taken at $\sqrt{2\times0\cdot7^2}=0''\cdot94$, or less than one second. Assuming, then, the probable error of one of LACAILLE's positions to be $2''$, and that of one of JOHNSON's (as above shown) to be $1''$, the probable error of the difference of the catalogues becomes $\sqrt{4+1}=2''\cdot236$; which divided by 80, the interval between the epochs, gives $0''\cdot028$ as the probable error of the annual proper motion deduced from the comparison of the two catalogues, so far as it depends on errors of observation. Hence it appears that a proper motion amounting to $0''\cdot1$ annually (the smallest which has been admitted in the present inquiry) considerably exceeds the probable errors of the catalogues, and consequently that the proper motions which have been under consideration not only have a real existence, but are determined with sufficient precision to give a result worthy of considerable confidence.

On the whole it may be said, that although the present result, if it stood by itself, would scarcely be considered as of sufficient weight to establish the fact and direction of the solar motion in space, yet coinciding as it does with those of ARGELANDER and OTTO STRUVE, it considerably increases the probability of the conclusions obtained by those astronomers. It is now shown that stars situated in every region of the heavens agree in their indication of a general motion directed towards one particular quarter; and as this agreement, not only of the results of different investigations, but of the great majority of the proper motions which have been ascertained and examined, cannot, on any reasonable supposition, be regarded as fortuitous, the inference is inevitable that they are, in part at least, systematic, and modified by the action of a general cause.

The proper motions which have been examined in this paper are not sufficiently numerous to warrant any speculation with respect to the nature of the path which the sun describes in space. Analogy leads us to infer that the sun must describe a curvilinear orbit, and if we suppose the orbit to be nearly circular, then the centre of motion will be situated in the plane passing through the sun perpendicular to the direction of his motion, and consequently in or near the great circle which has the point Q for one of its poles. The constellations through which this great circle passes are Piscis Australis, Pegasus, Andromeda, Perseus, &c. ARGELANDER, from various considerations, thought it probable that the sun's orbit is nearly in the plane of the Milky Way, and therefore that the central body must be sought for in this plane also. Now the two points of the sphere in which the great circle which is 90° from Q (as above determined) intersects the plane of the Milky Way, are situated, the one in Perseus, Æ $=49°$, Dec. $=+54\frac{1}{2}°$, and the other and diametrically opposite one between Lupus and the Southern Triangle. Near one of these two points, therefore, the central point of the sun's orbit must be situated, if both suppositions are correct; and ARGELANDER considers it most probable that the central point or body is in Perseus. MÄDLER, in a recent remarkable speculation, comes to the conclusion that the central sun is most probably situated in the Pleiades, and nearly in the direction of the star Alcyone (η Tauri) of that group. Perhaps the research is at present premature; but it seems not unreasonable to expect that a comparison of catalogues at the end of another half century will give the means of answering many interesting questions connected with the proper motions of the stars for the determination of which the data are still insufficient. It may then be possible to determine, for example, whether the apparent proper motions are uniform, or variable as has been supposed by POND and BESSEL; whether the direction of the sun's proper motion is gradually changing, or the apex maintains a fixed position in the heavens; whether the stars, which appear so irregularly grouped, form different independent systems, each having its own centre of attraction, or all obey the influence of one controlling force which pervades the visible universe. The solution of all these questions will, no doubt, be ultimately arrived at, but much yet remains to be done by the practical astronomer. Our knowledge of the proper motions of the southern stars is still very defective; and unless some other means are adopted than those which have yet been had recourse to, namely, the comparison of absolute places at distant epochs, a long time must elapse before the deficiency is supplied, and we may still say, in the words of HALLEY, that centuries may be required to discover the laws of the proper motions of the stars.

Method of Calculation.

The direction of the apparent motion of a star is conveniently defined by the angle which it makes with the circle of declination. Let S be the place of the star found by reducing the place given in the first catalogue to the epoch of the second, S′ its place given in the second catalogue, and P the north pole of the equator. Connecting these points by arcs of great circles, the arc SS′ represents the proper motion of the star in the interval between the epochs, and the angle PSS′ is the angle which has been denoted by ψ. This angle, which gives the direction of the star's motion, is reckoned from left to right all round the circle, from $\psi=0$ to $\psi=360°$, and is computed from the variations of right ascension and declination indicated by the comparison of the catalogues as follows :—

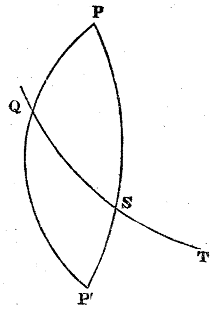


Let α and δ denote respectively the right ascension and declination of the star at the mean epoch (1790), and $\Delta\alpha$, $\Delta\delta$ be the annual variations of those quantities arising from proper motion ($\Delta\alpha$ being in seconds of arc), and Δs the annual variation of the star's place in the arc of a great circle, we have then

$$\left.\begin{aligned}\Delta s \sin \psi &= \cos \delta \Delta\alpha \\ \Delta s \cos \psi &= \Delta\delta \\ \tan \psi &= \frac{\cos \delta \Delta\alpha}{\Delta\delta}\end{aligned}\right\} \quad \ldots \ldots \ldots \ldots \quad (1.)$$

The values of ψ and Δs calculated from these formulæ are given, for all the stars under consideration, in the appended table.

To determine the direction of the parallactic motion, let Q be the point towards which the sun's motion is assumed to be directed, T the point diametrically opposite, S the place of a star, P, P′ the north and south poles of the equator respectively, and let PSP′, PQP′ and QST be great circles of the sphere. In consequence of the real motion of the sun towards Q, the star, as seen from the earth, will appear to move towards T, in the great circle QST, the position of which will be given in terms of the angle PST which it makes with the declination circle PSP′. Let the angle PST be denoted by ψ', and let A and D denote respectively the assumed right ascension and declination of Q.



The angle ψ' may be computed immediately from the formula

$$\cot \psi' = -\cos SP' \cot QP'S + \frac{\sin SP' \cot QP'}{\sin QP'S},$$

in which all the quantities are known, since SP′ is given in terms of δ, the star's declination, QP′ in terms of D, and the angle QPS represents the difference between α

and A, the right ascension of the star and of the point Q. It is more convenient, however, to compute the side QS (which is required in the subsequent calculations) in the first place, and to make use of it in computing ψ', because the same logarithms which are required in proceeding after this manner serve also for computing the coefficients of the equations of condition.

The ordinary trigonometrical formulæ give

$$\cos QS = \cos QP' \cos SP' + \sin QP' \sin SP' \cos QP'S.$$

Now since Q is assumed to be on the north side of the equator, and all the stars included in the present investigation are on the south side, we have

$$\cos QP' = \cos (90^\circ + D) = -\sin D; \quad \sin QP' = \sin (90^\circ + D) = +\cos D.$$
$$\cos SP' = \cos (90^\circ - \delta) = +\sin \delta; \quad \sin SP' = \sin (90^\circ - \delta) = +\cos \delta; \quad QP'S = \alpha - A.$$

Denoting QS by χ, and substituting these values in the above formula, we get

$$\cos \chi = -\sin D \sin \delta + \cos D \cos \delta \cos (\alpha - A). \quad \ldots \ldots \quad (2.)$$

Having found χ, or QS, the angle ψ' is computed from the formula

$$\sin \psi' = \frac{\sin QP' \sin QP'S}{\sin QS} = \frac{\cos D \sin (\alpha - A)}{\sin \chi}. \quad \ldots \ldots \quad (3.)$$

This sine belongs to two angles. In general there will be no difficulty with respect to the quadrant to which it belongs; but in a case of ambiguity, which may occur when ψ' is near 90°, recourse may be had to the formula for cot ψ' given above, which, on substituting for QP′, SP′, and QSP′ their expressions in terms of D, δ, and $(\alpha - A)$, becomes

$$-\cot \psi' = +\sin \delta \cot (\alpha - A) + \frac{\tan D \cos \delta}{\sin (\alpha - A)} \quad \ldots \ldots \quad (4.)$$

In computing from the above formulæ attention must be paid to the changes of sign of cos $(\alpha - A)$ and sin $(\alpha - A)$. The most convenient mode of proceeding, perhaps, is to take the stars in the order of right ascension, beginning at the declination circle, passing through Q, and adding 360° to all the values of α which are less than the assumed value of A, that is, to the right ascensions of all the stars excepting those which lie between the declination circle which passes through Q and that which passes through the first point of Aries. The values of $(\alpha - A)$ will thus be expressed in a series proceeding from 0° to 360°, and the sign to be prefixed to the cosine or sine becomes known from the value of the angle.

The equations of condition are formed as follows:—

Differentiating equation (4.) on the supposition that A and D are the variable quantities, we get

$$\frac{d\psi'}{\sin^2 \psi'} = \frac{1}{\sin (\alpha - A)} \left\{ \sin \delta + \tan D \cos \delta \cos (\alpha - A) \right\} dA + \frac{\cos \delta}{\sin (\alpha - A) \cos^2 D} dD;$$

now $\qquad \dfrac{\sin \psi'}{\sin (\alpha - A)} = \dfrac{\cos D}{\sin \chi}$, therefore

$$d\psi' = \frac{\cos D}{\sin^2 \chi} \left\{ \cos D \sin \delta + \sin D \cos \delta \cos (\alpha - A) \right\} dA + \frac{\cos \delta \sin (\alpha - A)}{\sin^2 \chi} dD. \quad \ldots \quad (5.)$$

If we now substitute for $d\psi'$ the difference between the angles ψ and ψ', or the value of $\psi-\psi'$ as found by equations (1.) and (3.), we shall have an equation in which dA and dD are the only unknown quantities. Every star furnishes a similar equation; and the values of dA and dD deduced from the whole of the equations by the method of least squares give the corrections to be applied to A and D, the assumed right ascension and declination of Q. Before this method can be applied, however, it is necessary to consider how the observations are affected by the situation and other circumstances of the individual stars, in order that all the equations may be reduced to the same degree of precision.

In the present inquiry it is assumed that the positions given in the catalogues, and the reductions from the first epoch to the second, are equally precise for all the stars; and in respect of the true proper motion, it has already been stated that we are not possessed of data to enable us to make any distinction between one star and another, and must therefore assume that, in this respect, all the equations have the same weight. Confining our consideration, therefore, to that part of the apparent motion which is caused by the displacement of the sun, the relative weights of the equations are determined as follows:—

Let the parallactic motions in right ascension and declination, in the unit of time (here assumed to be one year), be denoted by $\Delta\alpha$ and $\Delta\delta$ respectively, and the corresponding motion in the arc of a great circle by Δs, and we have the equations

$$\Delta s \sin\psi' = \cos\delta\Delta\alpha\,;\quad \Delta s\cos\psi' = \Delta\delta,$$

by differentiating which we get

$$d\Delta s\sin\psi' + \Delta s\cos\psi' d\psi' = d(\cos\delta\Delta\alpha),$$
$$d\Delta s\cos\psi' - \Delta s\sin\psi' d\psi' = d\Delta\delta,$$

whence

$$\Delta s d\psi' = \cos\psi' d(\cos\delta\Delta\alpha) - \sin\psi' d\Delta\delta.$$

Denoting in general the probable error of any quantity x by $\varepsilon(x)$, and its square by $\varepsilon^2(x)$, and observing that if $x=y\pm z$ the theory of probable errors gives

$$\varepsilon(x) = \sqrt{\{\varepsilon^2(y)+\varepsilon^2(z)\}},$$

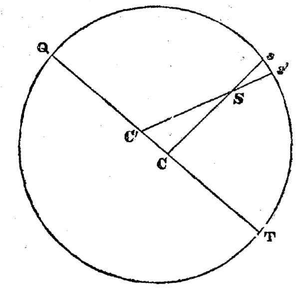
we have in respect of the above equation

$$\Delta s\,\varepsilon(\psi') = \sqrt{\{\cos^2\psi'\varepsilon^2(\cos\delta\Delta\alpha) + \sin^2\psi'\varepsilon^2(\Delta\delta)\}}.$$

Now the probable errors of $\Delta\alpha$ and $\Delta\delta$ depend manifestly on the precision and number of the observations from which the places of the stars have been determined; and as minute accuracy is not attainable in the present case, it may be assumed that the places of all the stars in each catalogue have been determined with equal precision. It may also be assumed that in respect of an equatorial star the probable error in right ascension is equal to the probable error in declination, and, generally, that $\varepsilon(\cos\delta\Delta\alpha)=\varepsilon(\Delta\delta)$. Denoting therefore the constant error by e, these two assumptions give $\varepsilon(\cos\delta\Delta\alpha)=\varepsilon(\Delta\delta)=e$, and the above equation becomes

$$\Delta s\,\varepsilon(\psi') = e\,;$$

whence it appears that the probable error of ψ' is inversely proportional to Δs.



To determine this quantity, let C be the place of the sun at the beginning, and C′ its place at the end of the time t, S the place of the star, and let straight lines joining CS and C′S meet the great circle whose plane contains the points C, C′, S, in s and s'. This plane also contains the points Q and T, which, therefore, are in the circumference of the same great circle. Now the angle CSC′, being the difference between QCS and QC′S, is the parallax, or angular variation of the apparent place of the star, which in consequence of the motion of the sun from C to C′ will appear to have moved from s to s', and is therefore (making $t=$ one year) the angle denoted by Δs. Hence the triangle CC′S gives

$$\sin \Delta s = \frac{\mathrm{CC'} \times \sin \mathrm{QC'S}}{\mathrm{CS}}.$$

Let CC′, which is constant, be denoted by R, and CS, the distance of the star, by r; then, observing that QC′S the angular distance of the star's place from Q is the angle which has been denoted by χ, and that $\sin \Delta s = \Delta s \sin 1''$, the above equation becomes

$$\Delta s = \frac{\mathrm{R} \sin \chi}{r \sin 1''}.$$

Substituting this in the equation $\Delta s\, \varepsilon(\psi') = e$, we have

$$\frac{\mathrm{R} \sin \chi}{r \sin 1''} \varepsilon(\psi') = e; \text{ or } \frac{\sin \chi}{r} \varepsilon(\psi') = \text{ a constant};$$

whence it follows that in order to reduce all the equations of condition to the same degree of precision, it is necessary to multiply each by $\sin \chi$, and to divide by a number proportional to r. The relative distances of the stars are however unknown, and in the present inquiry it has been assumed that they are all at the same mean distance, that is, r is assumed to be constant; and accordingly $\sin \chi$ becomes the measure of the precision of the equation.

Multiplying equation (5.) by $\sin \chi$, the formula for the equations of condition becomes

$$\sin \chi d\psi' = \frac{\cos \mathrm{D}}{\sin \chi} \left\{ \cos \mathrm{D} \sin \delta + \sin \mathrm{D} \cos \delta \cos (\alpha - \mathrm{A}) \right\} d\mathrm{A} + \frac{\cos \delta \sin (\alpha - \mathrm{A})}{\sin \chi} d\mathrm{D}; \quad . \quad . \quad (6.)$$

or, since

$$\frac{\cos \mathrm{D}}{\sin \chi} = \frac{\sin \psi'}{\sin (\alpha - \mathrm{A})},$$

$$\sin \chi d\psi' = \frac{\sin \psi}{\sin (\alpha - \mathrm{A})} \left\{ \cos \mathrm{D} \sin \delta + \sin \mathrm{D} \cos \delta \cos (\alpha - \mathrm{A}) \right\} d\mathrm{A} + \frac{\cos \delta \sin \psi'}{\cos \mathrm{D}} d\mathrm{D}. \quad . \quad . \quad (7.)$$

The double equation affords some advantage in checking the calculations; and it will be observed that the logarithms of all the sines and cosines required for computing the coefficients of dA and dD have already been used for computing χ and ψ'. It may be proper to state, that although in the appended table the logarithms of the sines and cosines of the different angles, and the coefficients of dA and dD in the equations of conditions, are given to four decimal places, all the calculations were made from the tables of five-figure logarithms.

The weights and probable errors of the values of dA and dD deduced from the equations of condition by the method of least squares, and the probable error of Ψ, or probable value of $(\psi-\psi')\sin\chi$, are computed according to the following formulæ:

Putting the above equation (7.) under the form

$$0=a dA+b dD-n,$$

and assuming, according to the usual notation, (aa) to denote the sum of the squares of the coefficient a, (bb) the sum of the squares of b, (nn) the sum of the squares of n, (ab) the sum of the products ab, and so on, and denoting also the *weights* of dA and dD respectively by $w(dA)$, $w(dD)$, the formulæ of the method of least squares give

$$dA=-\frac{(bb)(an)-(ab)(bn)}{(aa)(bb)-(ab)(ab)}\,;\quad dD=-\frac{(aa)(bn)-(ab)(an)}{(aa)(bb)-(ab)(ab)}$$

$$w(dA)=\frac{(aa)(bb)-(ab)(ab)}{(bb)}\,;\quad w(dD)=\frac{(aa)(bb)-(ab)(ab)}{(aa)}.$$

Let h denote the number of equations, $\varepsilon(\Psi)$ the probable error of Ψ, $\varepsilon(dA)$ and $\varepsilon(dD)$ the probable errors of dA and dD, and assume

$$(nn.2)=(nn)-\frac{(an)^2}{(aa)}-\frac{(bn.1)^2}{(bb.1)},$$

where

$$(bn.1)=(bn)-\frac{(ab)}{(aa)}(an)\,;\quad (bb.1)=(bb)-\frac{(ab)}{(aa)}(ab),$$

then the theory gives

$$\varepsilon(\Psi)=\cdot 67449\sqrt{\frac{(nn.2)}{h-2}}\,;\qquad \varepsilon(dA)=\frac{\varepsilon(\Psi)}{\sqrt{w(dA)}}\,;\qquad \varepsilon(dD)=\frac{\varepsilon(\Psi)}{\sqrt{w(dD)}}.$$

It may be remarked that in these formulæ $(nn.2)$ denotes the sum of the squares of the remaining errors when the values of dA and dD, found as above, are substituted in the equations of condition.

In the following table (which has been frequently referred to) the names of the stars are given, with their mean places for 1790, and the principal details of the calculation. The columns headed "LACAILLE — JOHNSON," "LACAILLE — HENDERSON," "BRADLEY — HENDERSON," contain the proper motions as given by Mr. JOHNSON and Prof. HENDERSON in their respective catalogues for the whole interval between the compared catalogues, those in Æ being in seconds of time. In the column of difference in Æ the positive sign shows that the right ascension is greater in the ancient catalogue than in the modern, and has consequently been diminished, and the negative that it has been increased, through the effect of the proper motion. In the column of difference in declination, the positive sign indicates a proper motion towards the south, and the negative a proper motion towards the north; the stars being all in the southern hemisphere, and their declinations consequently supposed to be affected with the negative sign.

No.	Star's name.	Magnitude.	α (1790).	δ (1790).	Lacaille — Johnson.		Lacaille — Henderson.	
					Æ.	Dec.	Æ.	Dec.
			° ′	° ′	s	″	s	″
1	η Pavonis	4·5	261 17·6	64 35·5	+ 0·24	+14·0	....	....
2	ι^1 Scorpii	3	263 13·8	40 1·4	+ 0·24	+ 6·6	+ 0·57	+ 6·1
3	β Telescopii	4	270 51·3	36 48·3	+ 0·86	+14·6	....	....
4	ε Sagittarii	3	272 33·6	34 27·8	....	....	+ 0·79	+15·4
5	ζ Pavonis	4	274 36·5	71 34·4	+ 0·28	+14·4	....	....
6	β^1 Sagittarii	3·4	286 52·6	44 49·9	— 0·36	+ 9·6	....	....
7	α Sagittarii	4·5	287 19·6	40 59·5	— 0·64	+12·4	....	....
8	ε Pavonis	4	293 59·8	73 26·1	— 1·35	+ 8·3	....	....
9	δ Pavonis	4	296 59·3	66 41·5	—16·28	+92·4	....	....
10	α Pavonis	2	302 13·8	57 23·4	— 0·79	+ 4·7	....	....
11	α Indi..............	3	305 40·8	48 0·4	— 0·92	— 3·8	....	....
12	γ Pavonis	3	317 12·6	66 17·9	— 1·21	—60·6	— 1·08	—60·7
13	γ Gruis	3	325 17·4	38 20·6	— 0·81	+ 7·1	....	....
14	α Gruis	2	328 43·7	47 58·2	— 1·26	+17·2	— 1·07	+15·3
15	β Octantis	5	335 50·2	82 28·3	+ 5·85	+ 0·7	+ 7·23	+ 0·6
16	β Gruis	3	337 30·6	47 58·5	— 1·65	+ 4·1	— 1·54	+ 4·6
17	α Piscis Aust. ...	1	341 30·2	30 43·8	....	....	— 1·93	+16·6
18	γ^1 Octantis	5	354 48·1	83 11·1	+10·51	+ 0·3	+11·62	+ 0·83
19	Toucan	5	2 15·4	66 6·6	—20·95	—95·9	....	....
20	β Hydri	3	3 36·9	78 26·3	—57·38	—23·1	—59·41	—24·3
21	α Phœnicis	2	3 58·0	43 26·9	— 1·33	+36·4	— 1·15	+36·8
22	η Phœnicis	5	8 28·0	58 36·7	+ 1·10	+42·3	....	....
23	γ Phœnicis	3·4	19 48·4	44 23·9	+ 0·09	+20·9	+ 0·33	+20·6
24	δ Phœnicis	4	20 37·3	50 10·1	— 1·23	— 9·6	....	....
25	χ Eridani	4	26 56·8	52 39·6	— 4·84	—25·2	....	....
26	α Hydri	3·4	28 2·3	62 35·8	— 3·00	— 1·6	— 2·75	— 1·4
27	β Reticuli	4	55 24·5	65 28·3	— 2·43	—12·0	....	....
28	52 Eridani	3	66 51·0	31 0·2	....	....	+ 0·69	+ 5·1
29	β Columbæ	3	85 53·1	35 51·5	+ 0·04	—32·7	+ 0·33	—31·0
30	α Equ. Pict.	4	101 30·5	61 43·1	+ 1·65	—24·4	....	....
31	σ Argûs............	4	110 38·7	42 53·1	+ 0·77	—10·6	....	....
32	γ^2 Argûs............	2	120 46·0	46 43·5	+ 0·63	+ 6·0	+ 0·86	+ 5·0
33	ε Argûs............	2	124 32·9	58 50·4	+ 0·03	— 7·3	+ 0·31	— 8·4
34	δ Argûs............	3	129 43·7	53 56·7	+ 0·21	+12·6	+ 0·39	+11·9
35	α Pisc. Vol.	4·5	134 46·5	65 33·7	— 0·65	+10·4	....	....
36	G in C Argûs...	5	136 7·4	71 45·1	+ 1·76	+43·5	....	....
37	β Argûs............	1·2	137 42·5	68 51·3	+ 2·61	— 7·0	+ 2·53	— 8·7
38	δ^2 Chamæl.........	5	160 54·8	79 26·0	+ 3·77	— 0·8	....	....
39	ε Chamæl.........	5	177 22·0	77 3·1	+ 3·24	+ 5·1	....	....
40	δ Centauri.........	3	179 23·3	49 33·1	+ 0·25	+ 7·9	+ 0·60	+ 7·3
41	δ Crucis............	3	181 1·6	57 34·9	+ 0·60	— 2·0	+ 0·99	— 3·5
42	β Chamæl.........	5	181 36·4	78 8·7	+ 3·24	+ 3·9	+ 3·79	+ 1·5
43	ε Crucis............	4	182 32·2	59 14·4	+ 2·01	—13·7	....	....
44	α^1 Crucis............	1	183 46·1	61 56·0	+ 1·64	— 5·9	+ 1·68	— 5·0
45	γ Crucis............	2·3	184 54·5	55 56·2	— 0·56	+15·3	— 0·19	+15·7
46	γ Muscæ	4	185 2·6	70 58·3	+ 1·36	+ 5·3	....	....
47	γ Centauri.........	3	187 30·3	47 48·2	+ 1·56	+ 6·6	+ 1·96	+ 5·1
48	ι Centauri.........	3	197 12·8	35 36·0	+ 1·85	+ 8·4	+ 2·33	+ 9·4
49	ε Centauri.........	3	201 40·6	52 23·4	+ 0·58	+ 8·6	+ 1·03	+ 7·6
50	3 Centauri.........	4·5	204 56·5	31 56·7	+ 0·83	+ 2·1	....	....
51	β Centauri.........	1	207 17·7	59 21·0	+ 0·51	+ 5·7	+ 0·95	+ 5·5
52	θ Centauri.........	3	208 35·9	35 19·7	+ 3·42	+45·7	+ 3·87	+45·1
53	δ Octantis	5	208 50·3	82 40·9	+ 8·00	+ 2·5	+ 9·12	+ 1·2
54	ι Lupi	4·5	211 30·7	45 4·7	— 0·96	+ 5·6	....	....
55	α^1 Centauri.........	4	216 21·8	59 57·6	+37·21	—65·3	+38·44	—68·5
56	α Lupi	3	217 0·9	46 28·4	+ 0·16	+ 8·2	+ 0·63	+ 6·5
57	β Lupi	3	221 12·9	42 16·4	— 0·04	+12·6	+ 0·42	+11·5
58	π Lupi	5	222 43·7	46 12·8	+ 0·84	+ 7·8	....	....
59	γ Triang. Aust....	3	224 23·9	67 53·0	+ 1·33	+ 2·8	+ 2·03	+ 3·0
60	β Triang. Aust....	3	234 12·2	62 45·6	+ 1·88	+33·4	+ 2·45	+32·8
61	ε Scorpii	3	249 8·9	33 53·6	....	....	+ 4·15	+23·8
62	η Scorpii	3·4	254 17·2	42 56·4	— 0·28	+25·1	— 0·04	+25·0
63	β Aræ	3	256 58·3	55 18·4	— 0·25	+ 9·4	....	....
64	δ Aræ	4	258 3·0	60 28·8	+ 0·44	+ 8·5	....	....
65	α Aræ	3	258 54·5	49 41·1	— 0·24	+ 9·0	....	....

No.	Log Δα.	Log Δδ.	Δs.	ψ.	ψ′.	ψ − ψ′.	Log sin χ.
1	8·6532	9·2430	0″·176	186° 17′·8	178° 42′·2	+ 7° 35′·6	9·9967
2	8·8692	8·8921	0·096	216 0·0	176 56·5	+ 39 3·5	9·9796
3	9·2075	9·2613	0·224	215 16·7	170 4·0	+ 45 12·7	9·9732
4	9·1546	9·2684	0·220	212 23·5	168 23·1	+ 44 0·4	9·9673
5	8·7202	9·2553	0·181	185 16·0	167 6·2	+ 18 9·8	9·9858
6	8·8293*n*	9·0792	0·129	158 15·2	157 6·7	+ 1 8·5	9·9948
7	9·0792*n*	9·1903	0·180	149 42·0	156 28·0	− 6 46·0	9·9900
8	9·4033*n*	9·0160	0·127	145 7·0	150 0·0	− 4 53·0	9·9772
9	0·4847*n*	0·0626	1·671	133 43·2	148 24·3	− 14 41·1	9·9885
10	9·1706*n*	8·7690	0·099	126 21·1	144 59·3	− 18 38·2	9·9967
11	9·2368*n*	8·6767*n*	0·125	67 37·8	142 42·7	− 75 4·9	0·0000
12	9·3243*n*	9·8718*n*	0·749	6 30·1	131 37·5	−125 7·4	9·9782
13	9·1815*n*	8·9482	0·149	126 41·2	129 44·4	− 3 3·2	9·9992
14	9·3321*n*	9·3003	0·246	144 14·1	126 36·0	+ 17 38·1	9·9915
15	0·1167	7·9026	0·172	267 19·8	108 21·8	+158 58·0	9·9358
16	9·4681*n*	8·7271	0·204	105 10·3	120 52·2	− 15 41·9	9·9824
17	9·5426*n*	9·3010	0·360	123 42·4	122 6·6	+ 1 35·8	9·9936
18	0·3086	7·3757	0·242	268 33·3	89 20·3	+179 13·0	9·9244
19	0·5942*n*	0·0787*n*	1·992	53 0·0	93 32·0	− 40 32·0	9·9165
20	1·0313*n*	9·4635*n*	2·175	82 18·8	83 43·5	− 1 24·7	9·9159
21	9·3591*n*	9·6524	0·479	159 43·3	106 51·2	+ 52 52·1	9·9317
22	9·3144	9·7233	0·540	191 29·0	93 30·0	+ 97 59·0	9·9033
23	8·5827	9·4061	0·256	186 7·5	96 54·0	+ 89 13·5	9·8666
24	9·3629*n*	9·0792*n*	0·190	50 54·7	90 10·0	− 39 15·3	9·8552
25	9·9579*n*	9·4983*n*	0·634	60 13·1	83 16·0	− 23 2·9	9·8304
26	9·7241*n*	8·2655*n*	0·244	85 40·6	71 54·0	+ 13 46·6	9·8431
27	9·6586*n*	9·1761*n*	0·241	51 35·1	36 10·0	+ 15 25·1	9·7705
28	9·0959	8·7885	0·123	240 6·3	101 11·0	+138 55·3	9·2838
29	8·5260	9·5923*n*	0·392	356 1·3	301 52·9	+ 54 8·4	9·0240
30	9·4905	9·4843*n*	0·338	334 19·9	324 20·3	+ 9 59·6	9·7291
31	9·1595	9·1222*n*	0·170	321 23·8	282 53·0	+ 38 30·8	9·6474
32	9·1360	8·8301	0·116	234 12·0	279 47·5	− 45 35·5	9·7493
33	8·4891	8·9833*n*	0·098	350 35·0	291 31·4	+ 59 3·6	9·8053
34	8·7398	9·1774	0·154	192 7·7	280 44·5	− 88 36·8	9·8177
35	9·0859*n*	9·1139	0·139	158 48·0	287 31·2	−128 43·2	9·8601
36	9·5185	9·7354	0·554	190 45·6	291 25·9	−100 40·3	9·8775
37	9·6751	8·9830*n*	0·196	299 23·3	287 23·7	+ 11 59·6	9·8745
38	9·8493	8·0000*n*	0·130	274 24·7	272 1·0	+ 2 23·7	9·9211
39	9·7836	8·8045	0·150	244 54·3	254 40·7	− 9 46·4	9·9380
40	8·8901	8·9701	0·106	208 21·2	241 16·6	− 32 55·4	9·9769
41	9·1635	8·5261*n*	0·085	293 15·7	242 41·9	+ 50 33·8	9·9689
42	9·8104	8·5239	0·137	255 52·5	251 17·0	+ 4 35·5	9·9403
43	9·5762	9·2336*n*	0·258	311 37·2	242 8·0	+ 69 29·2	9·9687
44	9·4851	8·8260*n*	0·159	294 59·0	242 7·5	+ 52 51·5	9·9665
45	8·8431*n*	9·2792	0·194	168 24·4	239 18·1	− 70 53·7	9·9763
46	9·4065	8·8212	0·106	231 27·0	244 40·0	− 13 13·0	9·9544
47	9·5097	8·8572	0·229	251 40·0	235 33·0	+ 16 7·0	9·9887
48	9·5847	9·0379	0·331	250 44·0	228 27·8	+ 22 16·2	0·0000
49	9·1687	8·9980	0·134	222 7·0	226 26·2	− 4 19·2	9·9948
50	9·1921	8·4191	0·135	258 45·5	224 2·5	+ 34 43·0	9·9964
51	9·1258	8·8373	0·097	224 44·5	223 3·5	+ 1 41·0	9·9911
52	9·8263	9·7461	0·781	224 27·5	221 26·7	+ 3 0·8	9·9968
53	0·1970	8·3591	0·202	263 29·7	227 42·5	+ 35 47·2	9·9471
54	9·2553*n*	8·8451	0·145	118 50·6	219 0·5	−100 9·9	9·9999
55	0·8427	9·9142*n*	3·581	283 15·1	215 56·8	+ 67 18·3	9·9945
56	8·8569	8·9562	0·103	208 43·3	214 57·5	− 6 14·2	9·9997
57	8·5340	9·1703	0·150	189 42·1	212 0·2	− 22 18·1	9·9965
58	9·1973	8·9890	0·146	228 11·1	210 38·5	+ 17 32·6	9·9987
59	9·4887	8·5511	0·121	252 57·2	210 10·2	+ 42 47·0	9·9875
60	9·5995	9·6089	0·445	204 6·7	201 32·2	+ 2 34·5	9·9963
61	9·8751	9·4575	0·686	245 16·2	189 42·8	+ 55 33·4	9·9645
62	8·4752*n*	9·4878	0·308	175 55·9	184 46·4	− 8 50·5	9·9861
63	8·6709*n*	9·0700	0·121	167 12·4	182 21·7	− 15 9·3	9·9997
64	8·9165	9·0263	0·114	200 56·2	181 27·2	+ 19 29·0	9·9994
65	8·6532*n*	9·0512	0·116	165 29·4	180 44·0	− 15 14·6	9·9960

No.	Star's name.	α (1790).	δ (1790).	Bradley — Henderson.	
				Æ.	Dec.
		° ′	° ′	s	″
66	β Ceti	8 15·6	19 8·5	−1·10	− 1·5
67	θ¹ Ceti	18 23·0	9 16·4	+0·41	+16·9
68	ζ Ceti	25 16·5	11 23·7	−0·05	+10·6
69	η Eridani	41 32·7	9 44·6	−0·47	+17·6
70	γ¹ Eridani	57 3·6	14 7·0	−0·37	+ 8·1
71	β Eridani	74 23·0	5 22·3	+0·46	+ 5·9
72	α Canis Maj.	98 58·3	16 26·6	+2·67	+97·2
73	15 Argûs	119 37·4	23 37·2	+0·41	− 5·9
74	δ Hydræ et Crat.	167 13·1	13 38·7	+0·53	−13·1
75	γ Corvi	181 15·8	16 22·6	+0·90	− 1·7
76	δ Corvi	184 45·2	15 20·7	+0·43	+12·1
77	β Corvi	185 51·0	22 19·0	+0·55	+ 5·7
78	α¹ Libræ	219 51·5	15 6·3	+0·51	+ 5·8
79	δ Ophiuchi	240 50·4	3 8·5	+0·32	+ 9·5
80	η Ophiuchi	254 36·6	15 27·0	−0·13	− 8·3
81	σ Sagittarii	281 31·3	26 32·5	−0·15	+ 7·6

No.	Log Δα.	Log Δδ.	Δs.	ψ.	ψ′.	ψ−ψ′.	Log sin χ.
			″	° ′	° ′	° ′	
66	9·3254n	8·2840n	0·201	84 30·2	117 41·5	− 33 11·3	9·9559
67	8·8968	9·3358	0·230	199 45·3	122 8·1	+ 77 37·2	9·9417
68	7·9830n	9·1332	0·136	176 2·0	122 2·9	+ 53 59·1	9·9085
69	8·9561n	9·3534	0·243	158 27·4	128 39·0	+ 29 48·4	9·8249
70	8·8522n	9·0164	0·125	146 23·8	135 28·6	+ 10 55·2	9·6669
71	8·9468	8·8788	0·116	229 20·6	170 9·2	+ 59 11·4	9·6656
72	9·7105	0·0956	1·340	201 33·8	223 45·1	− 22 11·3	9·6033
73	8·8968	8·8788n	0·105	316 19·0	246 42·5	+ 69 36·5	9·7697
74	9·0083	9·2252n	0·195	329 28·2	238 39·4	+ 90 48·8	9·9941
75	9·2382	8·3384n	0·167	277 28·6	235 45·0	+ 41 43·6	0·0000
76	8·9175	9·1907	0·174	207 12·4	234 45·2	− 27 32·8	9·9990
77	9·0244	8·8638	0·122	233 14·8	234 8·8	− 0 54·0	0·0000
78	8·9916	8·8713	0·120	231 15·5	218 13·1	+ 13 2·4	9·9419
79	8·7892	9·0856	0·136	206 46·3	205 15·2	+ 1 31·1	9·8071
80	8·3979n	9·0270n	0·109	12 45·6	185 50·5	−173 4·9	9·8724
81	8·4601n	8·9887	0·101	165 10·0	160 15·9	+ 4 54·1	9·9470

Equations of Condition.

From comparison with LACAILLE's Catalogue.

1	0=+·8434dA	+·0115dD	− 7·54
2	0=+·8425	+·0484	− 37·27
3	0=+·8321	+·1638	− 42·50
4	0=+·8269	+·1968	− 40·81
5	0=+·8405	+·0837	− 17·58
6	0=+·7971	+·3270	− 1·13
7	0=+·7878	+·3573	+ 6·61
8	0=+·8313	+·1690	+ 4·63
9	0=+·8176	+·2458	+ 14·30
10	0=+·7868	+·3666	+ 18·50
11	0=+·7397	+·4805	+ 75·08
12	0=+·7880	+·3563	+119·00
13	0=+·5896	+·7151	+ 3·05
14	0=+·6500	+·6373	− 17·29
15	0=+·8342	+·1474	−137·12
16	0=+·6174	+·6813	+ 15·07
17	0=+·4257	+·8632	− 1·57
18	0=+·8351	+·1406	−150·79
19	0=+·7403	+·4792	+ 33·44
20	0=+·8196	+·2362	+ 1·16
21	0=+·4782	+·8238	− 45·20
22	0=+·6642	+·6164	− 78·46
23	0=+·4564	+·8410	− 65·63
24	0=+·5488	+·7594	+ 28·12
25	0=+·5904	+·7141	+ 15·60
26	0=+·7210	+·5187	− 9·60
27	0=+·8071	+·2905	− 9·09
28	0=+·0630	−·9810	+ 26·71
29	0=+·4883	−·8169	− 5·72
30	0=+·7970	−·3275	− 5·36
31	0=+·4486	−·8469	− 17·10
32	0=+·5050	−·8009	+ 25·60
33	0=+·6926	−·5707	− 37·72
34	0=+·6140dA	−·6856dD	+ 58·24
35	0=+·7455	−·4678	+ 93·27
36	0=+·7921	−·3456	+ 75·94
37	0=+·7700	−·4081	− 8·98
38	0=+·8233	−·2173	− 2·00
39	0=+·8153	−·2562	+ 8·47
40	0=+·6227	−·6745	+ 31·22
41	0=+·6960	−·5648	− 47·07
42	0=+·8207	−·2307	− 4·00
43	0=+·7102	−·5361	− 64·66
44	0=+·7338	−·4391	− 48·94
45	0=+·6924	−·5710	+ 67·12
46	0=+·7903	−·3494	+ 11·90
47	0=+·6361	−·6567	− 15·70
48	0=+·5839	−·7216	− 22·27
49	0=+·7182	−·5243	+ 4·27
50	0=+·6028	−·6994	− 34·43
51	0=+·7683	−·4127	− 1·65
52	0=+·6479	−·6403	− 2·99
53	0=+·8381	−·1117	− 3·68
54	0=+·7169	−·5270	+100·15
55	0=+·7906	−·3484	− 66·46
56	0=+·7433	−·4677	+ 6·23
57	0=+·7467	−·4649	+ 22·12
58	0=+·7662	−·4181	− 17·49
59	0=+·8226	−·2243	− 41·57
60	0=+·8266	−·1992	− 2·53
61	0=+·8317	−·1661	− 51·19
62	0=+·8412	−·0722	+ 8·55
63	0=+·8431	−·0278	+ 15·14
64	0=+·8434	−·0148	− 19·46
65	0=+·8434	−·0098	+ 15·10

From comparison with BRADLEY's Catalogue.

66	0=+·1080dA	+·9918dD	+ 29·98
67	0=+·1138	−·9908	+ 67·87
68	0=+·1449	−·9851	+ 43·73
69	0=+·3448	−·9125	+ 19·92
70	0=+·4992	−·8062	+ 5·07
71	0=+·8264	−·2019	+ 27·41
72	0=+·5211	+·7864	− 8·90
73	0=+·0572	+·9978	+ 40·96
74	0=+·1502dA	−·9840dD	− 89·60
75	0=+·2872	−·9404	− 41·72
76	0=+·3020	−·9338	+ 27·48
77	0=+·3863	−·8890	+ 0·90
78	4=+·5955	−·7082	− 11·41
79	0=+·7280	−·5050	− 0·97
80	0=+·8377	−·1163	+129·57
81	0=+·7875	+·3581	− 4·34